1-1 Practice

Functions

Write each set of numbers in set-builder and interval notation, if possible.

1. \{-3, -2, -1, 0, 1, \ldots\}  
   \{x \mid x \geq -3, x \in \mathbb{Z}\}

2. \{-6.5 < x \leq 3\}  
   \{x \mid -6.5 < x \leq 3, x \in \mathbb{R}\}; (-6.5, 3]

3. all multiples of 2  
   \{x \mid x = 2n, n \in \mathbb{Z}\}

4. \{x \mid x < 0 \text{ or } x > 8, x \in \mathbb{R}\}; (-\infty, 0) \cup (8, \infty)

Determine whether each relation represents \(y\) as a function of \(x\).

5. The input value \(x\) is a car's license plate number, and the output value \(y\) is the car's make and model. \textbf{function}

6. \textbf{not a function}

7. \textbf{not a function}

8. \(-x + y = 3x\)  
   \textbf{function}

9. \(x = 5(y - 1)^2\)  
   \textbf{not a function}

Find each function value.

10. \(h(x) = x^2 - 8x + 1\)
    a. \(h(-1) = 10\)
    b. \(h(2x) = 4x^2 - 16x + 1\)
    c. \(h(x + 8) = x^2 + 8x + 1\)

11. \(f(a) = -3\sqrt{a^2 + 9}\)
    a. \(f(4) = -15\)
    b. \(f(3a) = -9\sqrt{a^2 + 1}\)
    c. \(f(a + 1) = -3\sqrt{a^2 + 2a + 10}\)

State the domain of each function.

12. \(g(x) = \sqrt{-3x - 2}\)
    \(\{x \mid x \leq -\frac{2}{3}, x \in \mathbb{R}\}\)

13. \(h(t) = \frac{2t - 6}{t^2 + 6t + 9}\)
    \(\{t \mid t \neq -3, t \in \mathbb{R}\}\)

14. Find \(f(-4)\) and \(f(11)\) for the piecewise function \(f(x) = \begin{cases} 3x^2 + 16 & \text{if } x < -2 \\ \sqrt{x - 2} & \text{if } -2 \leq x \leq 11 \\ -75 & \text{if } x > 11 \end{cases}\)
    64; 3
Practice 1-1

10. \( h(-1) = (-1)^2 - 8(-1) + 1 \)
\[ h(2x) = (2x)^2 - 8(2x) + 1 \]
\[ = 1 + 8 + 1 = 10 \]
\[ = 4x^2 - 16x + 1 \]

\( h(x+8) = (x+8)^2 - 8(x+8) + 1 \)
\[ (x+8)(x+8) - 8(x+8) + 1 \]
\[ x^2 + 16x + 64 - 8x - 64 + 1 \]
\[ = x^2 + 8x + 1 \]

11. \( f(4) = -3 \sqrt{4^2 + 9} \)
\[ f(3a) = -3 \sqrt{(3a)^2 + 9} \]
\[ = -3 \sqrt{16 + 9} \]
\[ = -3 \sqrt{25} \]
\[ = -3 \times 5 \]
\[ = -15 \]

\[ f(a+1) = -3 \sqrt{(a+1)^2 + 9} \]
\[ = -3 \sqrt{(a+1)(a+1) + 9} \]
\[ = -3 \sqrt{a^2 + 2a + 1 + 9} \]
\[ = -3 \sqrt{a^2 + 2a + 10} \]

12. \( g(x) = \sqrt{-3x-2} \)
\[-3x-2 \geq 0 \]
\[-3x \geq 2 \]
\[ x \leq -\frac{2}{3} \]
\( \{ x \mid x \leq -\frac{2}{3}, x \in \mathbb{R} \} \)

13. \( h(t) = \frac{2t-6}{t^2 + 6t + 9} \)
\[ = \frac{2t-6}{(t+3)(t+3)} \neq 0 \]
\[ t+3 \neq 0 \]
\[ t \neq -3 \]
\( \{ t \mid t \neq -3, t \in \mathbb{R} \} \)

14. \( f(-4) = 3(-4)^2 + 16 \)
\[ = 3 \times 16 + 16 \]
\[ = 64 \]
\( f(11) = \frac{\sqrt{11-2}}{\sqrt{9}} \)
\[ = \frac{\sqrt{9}}{3} \]
\[ = 3 \]
1. Use the graph of the function shown to estimate $f(-2.5)$, $f(1)$, and $f(7)$. Then confirm the estimates algebraically. Round to the nearest hundredth, if necessary.

12; 5; 9

Use the graph of $h$ to find the domain and range of each function.

2. $D = [-4, 3]$, $R = [-6, 5]$

3. $D = (-\infty, 4]$, $R = (-\infty, 3]$

4. Use the graph of the function to find its $y$-intercept and zeros. Then find these values algebraically. $y$-int: $-8$, zero: 2;

Use the graph of each equation to test for symmetry with respect to the $x$-axis, $y$-axis, and the origin. Support the answer numerically. Then confirm algebraically.

5. origin; $-y = \frac{-2}{-x}$, $y = \frac{-2}{x}$

6. $y$-axis; $y = -0.5(-x)^2 - 3$, $y = -0.5(x)^2 - 3$

7. Graph $g(x) = \frac{1}{x^2}$ using a graphing calculator. Analyze the graph to determine whether the function is even, odd, or neither. Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.

even; $f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$; symmetric with respect to the $y$-axis
Practice 1-2

4. Zeros: \( y = 0 \)
\[ 0 = 4 \sqrt[3]{x-1} + 4 \]
\[ 4 = 4 \sqrt[3]{x-1} \]
\[ \frac{4}{4} \]
\[ 1 = x-1 \]
\[ 2 = x \]

\[ y = -8 \]

5-6 Support the answer numerically means make a table!

5. \[
\begin{array}{c|c}
 x & y \\
 2 & -4 \\
 -2 & 4 \checkmark \\
 4 & -2 \checkmark \\
 -4 & 2 \checkmark \\
\end{array}
\]

6. \[
\begin{array}{c|c}
 x & y \\
 4\Box & -2 \\
 -4 & -2 \checkmark \\
\end{array}
\]
**1-3 Practice**

**Continuity, End Behavior, and Limits**

Determine whether each function is continuous at the given x-value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as infinite, jump, or removable.

1. \( f(x) = -\frac{2}{3x^2} \); at \( x = -1 \)

2. \( f(x) = \frac{x}{x + 4} \); at \( x = -4 \)

3. \( f(x) = x^3 - 2x + 2 \); at \( x = 1 \)

4. \( f(x) = \frac{x + 1}{x^2 + 3x + 2} \); at \( x = -1 \) and \( x = -2 \)

Determine between which consecutive integers the real zeros of each function are located on the given interval.

5. \( f(x) = x^3 + 5x^2 - 4 \); \([-6, 2]\)

6. \( g(x) = x^4 + 10x - 6 \); \([-3, 2]\)

\([-5, -4], [-1, 0], [0, 1]\) \([-3, -2], [0, 1]\)

Use the graph of each function to describe its end behavior. Support the conjecture numerically.

7. ![Graph of f(x) = -6x / (3x - 5)]

8. ![Graph of f(x) = x^2 - 4x - 5]

\[ \lim_{x \to -\infty} f(x) = -2; \quad \lim_{x \to \infty} f(x) = -2; \quad \lim_{x \to -\infty} f(x) = \infty; \quad \lim_{x \to \infty} f(x) = \infty \]

9. **ELECTRONICS** Ohm's Law gives the relationship between resistance \( R \), voltage \( E \), and current \( I \) in a circuit as \( R = \frac{E}{I} \). If the voltage remains constant but the current keeps increasing in the circuit, what happens to the resistance? **Resistance decreases and approaches zero.**
Practice 1-3

1. Domain: \(3x^2 \neq 0\)
   \[
   x^2 \neq 0 \\
   x \neq 0
   \]
   \((-\infty, 0) \cup (0, \infty)\)
   Since \(x = -1\) is in the domain of \(f(x)\), the function is continuous at \(x = -1\).

2. Domain: \(x + 4 \neq 0\)
   \[
   x \neq -4 \quad (-\infty, -4) \cup (-4, \infty) \\
   \]
   Since \(x = -4\) is not in the domain of \(f(x)\), the function is not continuous at \(x = -4\).
   \[
   \begin{array}{c|c|c|c|c|c|c|c}
   x & -4.1 & -4.01 & -4.001 & -4 & -3.999 & -3.99 & -3.9 \\
   f(x) & 61 & 60.1 & 60.01 & 60 & -59.99 & -59.9 & -59 \\
   \end{array}
   \]
   \[
   \lim_{{x \to -4^-}} f(x) \to \infty \quad \lim_{{x \to -4^+}} f(x) \to -\infty
   \]
   Therefore, the function has infinite discontinuity at \(x = -4\).

3. Domain: \((-\infty, \infty)\)
   Since \(x = 1\) is in the domain of \(f(x)\), the function is continuous at \(x = 1\).
4. Domain: \( x^2 + 3x + 2 \neq 0 \)
\[
(x+2)(x+1) \neq 0
\]
\[
x+2 \neq 0 \quad x+1 \neq 0
\]
\[
x \neq -2 \quad x \neq -1
\]
\[
(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)
\]

Since neither \( x = -2 \) or \( x = -1 \) are in the domain of \( f(x) \), the function is not continuous at either both.

<table>
<thead>
<tr>
<th>(-2.1)</th>
<th>(-2.11)</th>
<th>(-2.01)</th>
<th>(-2)</th>
<th>(-1.99)</th>
<th>(-1.99)</th>
<th>(-1.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-10)</td>
<td>(-100)</td>
<td>(-1000)</td>
<td>(1000)</td>
<td>(100)</td>
<td>(10)</td>
<td></td>
</tr>
</tbody>
</table>

\[
\lim_{{x \to -2^{-}}} f(x) \to -\infty \quad \lim_{{x \to -2^{+}}} f(x) \to \infty
\]

Therefore, there is an infinite discontinuity at \( x = -2 \).

<table>
<thead>
<tr>
<th>(-1.1)</th>
<th>(-1.01)</th>
<th>(-1.001)</th>
<th>(-1)</th>
<th>(-0.99)</th>
<th>(-0.99)</th>
<th>(-0.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1.11)</td>
<td>(-1.011)</td>
<td>(-1.0001)</td>
<td>(0.99)</td>
<td>(0.99)</td>
<td>(0.9909)</td>
<td>(0.90909)</td>
</tr>
</tbody>
</table>

\[
\lim_{{x \to -1}} f(x) = 1
\]

Therefore, there is a removable discontinuity at \( x = -1 \).
1-4 Practice

Extrema and Average Rates of Change

Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Support the answer numerically.

1. \( f(x) = x^2 - 2x^2 - 2x^3 \)
   - increasing on \((-\infty, 0)\);
   - decreasing on \((0, 1.5)\);
   - increasing on \((1.5, \infty)\);

2. \( f(x) = \frac{2x}{x^2} \)
   - decreasing on \((-\infty, 0)\);
   - decreasing on \((0, \infty)\);

Estimate to the nearest 0.5 unit and classify the extrema for the graph of each function. Support the answers numerically.

3. \( f(x) = x^4 - 3x^2 + x - 5 \)
   - rel. min. of \(-8.5\) at \(x = -1.5\);
   - rel. max. of \(-5\) at \(x = 0\);
   - rel. min. of \(-6\) at \(x = 1\);

4. \( f(x) = x^3 + x^2 - x \)
   - rel. max. of \(1\) at \(x = -1\);
   - rel. min. of \(0\) at \(x = 0.5\);

5. GRAPHING CALCULATOR Approximate to the nearest hundredth the relative or absolute extrema of \( h(x) = x^5 - 6x + 1 \). State the \(x\)-values where they occur.
   - rel. max. \((-1.05, 6.02)\);
   - rel. min. \((1.05, -4.02)\)

Find the average rate of change of each function on the given interval.

6. \( g(x) = x^3 + 2x^2 - 5 \); \([-4, -2]\)
   - \(-132\)

7. \( g(x) = -3x^3 - 4x \); \([2, 6]\)
   - \(-160\)

8. PHYSICS The height \(t\) seconds after a toy rocket is launched straight up can be modeled by the function \( h(t) = -16t^2 + 32t + 0.5 \), where \( h(t) \) is in feet. Find the maximum height of the rocket. \(16.5\) ft